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A new locally linear embedding scheme in light of  
Hessian eigenmap  
  
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Abstract  
  
We provide a new interpretation of Hessian locally linear embedding (HLLE),  
revealing that it is essentially a variant way to implement the same idea of locally  
linear embedding (LLE). Based on the new interpretation, a substantial simpliﬁcation  
can be made, in which the idea of “Hessian” is replaced by rather arbitrary weights.  
Moreover, we show by numerical examples that HLLE may produce projection-like  
results when the dimension of the target space is larger than that of the data manifold,  
and hence one further modiﬁcation concerning the manifold dimension is suggested.  
Combining all the observations, we ﬁnally achieve a new LLE-type method, which  
is called tangential LLE (TLLE). It is simpler and more robust than HLLE.  
  
1 Introduction  
  
i=1 be a collection of data points in some RD. The goal of nonlinear dimen-  
Let X = {xi}N  
sionality reduction (or manifold learning) is to ﬁnd for X a representation Y = {yi}N  
i=1  
in some lower dimensional Rd, under the assumption that X lies on some unknown sub-  
manifold M in RD.  
  
Among the several existing manifold learning methods, Hessian eigenmap [2], also  
called Hessian locally linear embedding (HLLE), is one that exhibits prominent perfor-  
mance on the popular synthetic data “Swiss roll with a hole”. It can be regarded as a  
generalization of Laplacian eigenmap [1] in some respect or LLE [4] in another. How-  
ever, its procedure concerning the construction and minimization of “Hessian” is much  
more sophisticated.  
  
In this paper, we will provide a new interpretation of the mechanism behind HLLE,  
revealing that what it really does follows the same idea as LLE: Asking Y to satisfy  
the local linear relations for X as best as possible. The main differences lie in their  
ways of describing the local linear relations. Roughly speaking, HLLE only ﬁts local  
  
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